A Nonlinear Exponential ARX Model Based Multivariable Generalized Predictive
Control Strategy for Thermal Power Plants

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Abstract: This paper presents a modeling and control method for thermal power plants having nonlinear
dynamics varying with load. First a load-dependent exponential ARX (Exp-ARX) model that can effectively
describe the plant nonlinear properties and requires only off-line identification is presented. The model is then
used to establish a constrained multivariate multi-step predictive control (ExpMPC) strategy whose
effectiveness is illustrated by a simulation study of a 600 megawatt (MW) thermal power plant. Although the
predictive control algorithm may be used without resorting to online parameter estimation, it is much more
reliable, and displays much better control performance than the usual GPC algorithm.

Keywords: power systems, nonlinear systems, ARX models, system identification, constraints, predictive
control.

Nomenclature: MWD: Megawatt demand or load demand, i.e. $M(t)$;

STE: Superheater outlet steam temperature error, i.e. $y_3(t) = K_{MSB} x_{33}$;

RTE: Reheater outlet steam temperature error, i.e. $y_3(t) = K_{RHb} x_{15}$;

TPE: Main steam pressure error, i.e. $y_3(t) = x_5$;

SP1: Primary spray attemperator;

SP2: Secondary spray attemperator;
FF: Fuel flow rate;  
FW: Feedwater flow rate;  
RGD: Opening of the reheater flue gas damper.

1. Introduction

In normal use, thermal power plants display daily load fluctuations which can be anything from 30% to 100% of full load. The plant must be capable of operating as a controlled system across a wide range of working conditions. Since the relationships governing the main controlled variables (main steam temperature, reheated steam temperature, and main steam pressure) depend to a large extent on load, the behavior of the plant becomes quite non-linear. However, at present, the multi-loop PID controller remains the most widely used method in the process control.

Over the last 10 years, several advanced control methods for the control of thermal power plants have been reported. Examples are the generalized predictive control (GPC) method based on local model networks [5], the self-tuning minimum variance control method for steam temperature control [3], the self-tuning multi-loop generalized predictive control (GPC) method [2], and the multivariable dynamic matrix control (DMC) method based on linear models [6]. Of these, the local model network based GPC uses a set of local linear models to describe the non-linearity of the plant. However, in order to represent plant non-linearity satisfactorily, the required number of local linear models often becomes too large to be implemented easily. The self-tuning GPC, DMC and minimum variance control methods all require on-line parameter estimation to deal with plant non-linearity. In real-time, the convergence and computational cost of on-line parameter estimation poses considerable problems.

Toyoda [7] introduced an algorithm based on an amplitude-dependent auto-regressive model [1], which uses an off-line method to identify the load-dependent non-linearity of thermal power plants. However, in Toyoda's method, the effects of stochastic process disturbances are not taken into account. Furthermore, the cost function of the controller is not very appropriate, since it does not involve the outputs or the state errors, and on-line optimization has to be used to compute control signals based on a nonlinear model and non-linear programming techniques. In this case, sometimes it is impossible to obtain a feasible solution, and control may become divergent.

In this paper, we also follow Toyoda's method [7] of using an (amplitude dependent) exponential auto-regressive model [1], thus obviating the problems resulting from on-line parameter estimation. Like the model used by Toyoda [7], our model is an Exp-ARX model depending on load as an exogenous variable. However, noting that the dynamics of the thermal plant system are different under situations of increasing or decreasing load, our
model also depends on the incremental change in load. As will be shown in section 3, the model effectively represents the nonlinear dynamics of the system over a wide range of operating conditions.

In order to deal with the controller cost function problems noted in the use of Toyota’s method [7], we have also built a new type of multivariate multi-step-ahead predictor. Using this nonlinear predictor, a constrained multivariate long-range predictive control strategy has been designed. In section 5, the modeling and control methods proposed in this paper are implemented in a simulation study of a 600MW fossil-fired thermal power plant using MATLAB SIMULINK (The MathWorks, Inc.).

2. Thermal power plant non-linearity

This paper will use a thermal power plant simulation of a 600MW sliding pressure type boiler-turbine system [8-9] created in MATLAB SIMULINK as the plant to be controlled. This simulation model is based on a set of physical models designed in accordance with operational and physical data of an actual plant and includes a total of 40 variables. The model is not linearized near equilibrium, and can imitate the strong nonlinear dynamic properties of power systems under large load variations. In the simulation model, sets of conventional multi-loop gain-scheduling PID regulators are also used to control the power plant as would be common in practice. The details of the simulation model may be described by a set of differential equations given in Appendices A and B [8]. The gain-scheduling PID controllers and the constrained multivariable generalized predictive controller (ExpMPC) presented in this paper will operate in parallel in the proposed control system. This is because the actual plant will commonly already be working with PID controllers, so parallel operation will obviate the need to drastically change the structure of the actual control system, and will also guarantee operational safety. Therefore the plant to be controlled by the ExpMPC consists of the turbine-boiler system and the multi-loop PID controllers. The outputs and the inputs of the controlled plant are given as follows

\[
Y(t) = \begin{bmatrix}
    y_1(t) & \text{STE (main steam temperature error / °C)} \\
    y_2(t) & \text{RTE (reheat steam temperature error / °C)} \\
    y_3(t) & \text{TPE (main steam pressure error / bar)}
\end{bmatrix}
\]

\[
U(t) = \begin{bmatrix}
    u_1(t) & \text{FFI (fuel flow increment)} \\
    u_2(t) & \text{1SPI (1st stage spray flow increment)} \\
    u_3(t) & \text{2SPI (2st stage spray flow increment)} \\
    u_4(t) & \text{GRI (flue gas recirculation flow increment)} \\
    u_5(t) & \text{TCI (turbine control increment)}
\end{bmatrix}
\]

here, the unit of \( U(t) \) is the open width of the damper (expressed as a percentage), and the sampling interval is 30 seconds. For safe and economical operation of the plant, the errors of the output variables \( Y(t) \) should be controlled to be as near to zero as possible, whatever the load level. The non-linearity of the controlled plant may be seen from its step responses. Figs. 1 and 2 are unit-step responses to \( u_3(t) \) and \( u_4(t) \) when the load demands are 25%, 50%, 75% and 90%MWD (Megawatt Demand) of the full load respectively.

Figs. 1 and 2 show that the dominant time constants are over 10 minutes, so a sampling interval of 30 seconds is
quite appropriate for digital control. The steady-state gain and step-response sequences are varying acutely with load. The non-linear characteristics are mainly due to variation in plant static gains with loads. This provides a justification for using a load-dependent Exp-ARX model to describe the nonlinear dynamics of thermal power plants.

3. Modeling of the controlled system

In the single variable case, the exponential auto-regressive model [1] may be written as follows

$$x_t = \sum_{i=1}^{p} (\varphi_i + \pi_i e^{-\gamma t^i_i})x_{t-i} + e_t$$  \hspace{1cm} (1)$$

where $x_t$ is the state, $\varphi_i, \pi_i$ are constants, $\gamma$ is a scaling parameter, and $e_t$ is white noise. The idea of the model is that it makes the instantaneous characteristic roots of the AR model depend on state amplitude. The model displays certain well-known features of nonlinear vibration theory, such as amplitude-dependent frequency, jump phenomena and limit cycle behavior.

For thermal power plants, an unsophisticated method of describing the dynamics could be by using different linear ARX models at certain fixed load levels or operating points, where non-linearity will result from the power load-cycling operations. Extending this idea, a more flexible description of the change in dynamics could be provided by a time-varying ARX model, such as the Exp-ARX model below, whose parameters change with load demand. Such a model can represent the dynamic behavior of the system over the whole operating range, and may be estimated off-line with only slightly more computational effort than fitting a single linear ARX model. This load-dependent multi-variable Exp-ARX model may be constructed as follows

$$A_t(q^{-1})Y(t) = B_t(q^{-1})U(t-1) + D_t(q^{-1})\Delta M(t-1) + T(q^{-1})\xi(t)$$  \hspace{1cm} (2)$$

where $Y(t) \in \mathbb{R}^n$, $U(t-1) \in \mathbb{R}^n$, $M(t-1)$ is the load
demand, and $\Delta M(t-1)$ is the load demand increment, \(\xi(t) \in \mathbb{R}^n\) denotes the noise sequence which is assumed to satisfy
\[
E\{\xi(t)F_t\} = 0, \quad E\{\xi(t)\xi(t)^T\} = \Omega
\]
where $F_t$ denotes the $\sigma$-algebra generated by the data up to and including time $t$, and $\Omega$ is a positive definite matrix. In (2), $A_i(q^{-1})$, $B_i(q^{-1})$, and $D_i(q^{-1})$ are the $n \times n$, $n \times m$ and $n \times 1$ load-dependent polynomial matrices in the unit delay operator $q^{-1}$ respectively, $T(q^{-1})$ is the $n \times n$ Hurwitz design polynomial matrix, which is used to represent prior knowledge about the process noise, because successful identification of the actual noise model structure which is multi-source and time-varying is unlikely. In (2), take
\[
\begin{align*}
A_i(q^{-1}) &= I + A_{i1}q^{-1} + \cdots + A_{ik}q^{-ka} \\
B_i(q^{-1}) &= B_{i0} + B_{i1}q^{-1} + \cdots + B_{ik}q^{-kb} \\
D_i(q^{-1}) &= D_{i0} + D_{i1}q^{-1} + \cdots + D_{ik}q^{-kd} \\
T(q^{-1}) &= I + T_{1}q^{-1} + \cdots + T_{k}q^{-kt}
\end{align*}
\]
and
\[
\begin{align*}
A_{ij} &= \Phi_{ai}(0) + \Phi_{ai}(1)e^{-\lambda_i\|M(t-1)-M_0\|}, \\
B_{ij} &= \Phi_{bi}(0) + \Phi_{bi}(1)e^{-\lambda_i\|M(t-1)-M_0\|}, \\
D_{ij} &= \Phi_{di}(0) + \Phi_{di}(1)e^{-\lambda_i\|M(t-1)-M_0\|}
\end{align*}
\]
where $\|\|$ denotes a vector norm, $\lambda_a, \lambda_b$ and $\lambda_d$ are the scaling parameters, $Q_a, Q_b$ and $Q_d$ are $2 \times 2$ weighting matrices, $M_0$ and $\Delta M_0$ are the datum load and its increment respectively. In the exponential factors of (4), $M(t-1)$ and $\Delta M(t-1)$ respectively represent the messages of ‘position’ and ‘velocity’ of the system operating point form which the parameters of model (2) track the load variations.

The estimation procedure for the Exp-ARX model is computationally simple, involving only the use of the least squares methods to estimate the coefficients $\Phi_{ai}(f)(x = a,b,d; j = 0,1)$ after first fixing the model orders $ka, kb, kd, k_t$ and the parameters $\lambda_a, \lambda_b, \lambda_d, Q_a, Q_b, Q_d, M_0, \Delta M_0$. The Akaike Information Criterion (AIC) may be used to choose the model orders [4]. Generally, $\lambda_a, \lambda_b, \lambda_d, Q_a, Q_b$ and $Q_d$ may be chosen as follows
\[
\begin{align*}
\lambda_i &= -\log(\alpha_j) \left\| \frac{M(t) - M_0}{\Delta M(t)} \right\|_2, \\
Q_i &= \text{diag}(\beta_{i1}, \beta_{i2}), \quad i = a,b,d, \\
&\quad \beta_{i1} = 1, \beta_{i2} = 100 \sim 500, \alpha_j \in [0.1 \sim 0.00001].
\end{align*}
\]
In this case, the model orders and other parameters used were
\[
ka = 8, kb = 14, kd = 14, T(q^{-1}) = I, \\
Q_a = Q_b = Q_d = \text{diag}(1,300), \quad M_0 = 20(\%), \\
\Delta M_0 = 0, \lambda_a = 0.0001, \lambda_b = \lambda_d = 0.1
\]
Fig.3 shows the root loci of the simulation plants represented by the fitted Exp-ARX model, and illustrates how the model dynamics vary with load. It also reveals that the model has an ability to represent the non-linear dynamics of the plant due to working point changes. Fig. 4 shows a comparison between the actual data and data generated by the fitted Exp-ARX model, where the plant inputs were a set of independent PRBS signals with unit amplitude, from which it is clear that the Exp-ARX model provides a very close fit to the actual data.
4. Multi-step predictive control

The model based multi step predictive control method has various advantages, such as rolling optimization, multi-step-ahead prediction and feedback correction etc. Here, an Exp-ARX model based multi-step predictive control algorithm (ExpMPC) will be presented for thermal power plant control.

4.1 Multi-step-ahead prediction

Theorem 1: Multivariable nonlinear predictor

Consider the systems described by (2)-(4). If we can obtain

\[
\{ M(t+j-1), \Delta M(t+j-1) | j = 1, 2, \ldots, N \}
\]

and constrain \( \{ U(t+j-1) | j = 1, 2, \ldots, N \} \) to be \( F_j \)-measurable, then \( j(j=1,2,\ldots,N) \) step ahead optimal predictive outputs are

\[
\hat{Y}(t+j|t) = E\{ Y(t+j)|F_1 \} = G_j(q^{-1})U(t+j-1) + Y_0(t+j) \quad \text{(5)}
\]

where

\[
Y_0(t+j) = T(q^{-1})^{-1}F_j(q^{-1})Y(t) + T(q^{-1})^{-1}H_j(q^{-1})U(t-1)
\]

\[
+ T(q^{-1})^{-1}E_j(q^{-1})D_{r_j}(q^{-1})\Delta M(t+j-1) \quad \text{(6)}
\]

\( E_j(q^{-1}), F_j(q^{-1}), G_j(q^{-1}) \), and \( H_j(q^{-1}) \) are the solutions of two Diophantine equations

\[
T(q^{-1}) = E_j(q^{-1})A_{s,j}(q^{-1}) + q^{-j}F_j(q^{-1}) \quad \text{(7)}
\]

\[
E_j(q^{-1})B_{s,j}(q^{-1}) = T(q^{-1})G_j(q^{-1}) + q^{-j}H_j(q^{-1}) \quad \text{(8)}
\]

Proof: Multiply (7) by \( Y(t+j) \) and introduce (2) and (8) into the resulting expression. This yields

\[
Y(t+j) = G_j(q^{-1})U(t+j-1) + Y_0(t+j) + \hat{V}_j(q^{-1})\xi(t+j) + \hat{V}_j(q^{-1})\xi(t) \quad \text{(9)}
\]

here

\[
\hat{V}_j(q^{-1})q^j + \hat{V}_j(q^{-1}) = T(q^{-1})^{-1}E_j(q^{-1})T(q^{-1})q^j
\]

\[
\hat{V}_j(q^{-1}) = \hat{V}_{j_1} + \hat{V}_{j_2}q^{-1} + \cdots + \hat{V}_{j_{j-1}}q^{-j+1}
\]

If require \( \{ U(t+j-1) | j = 1, 2, \ldots, N \} \) to be \( F_j \)-measurable and ignore the random disturbance term \( \hat{V}_j(q^{-1})\xi(t) \), then (9) implies (5). \( \square \)

Theorem 2: The recursive solution of the Diophantine equation (7) may be achieved as follows

\[
E_{j+1}(q^{-1}) = E_j(q^{-1}) + q^{-j}F_j(q^{-1}), j = 1, 2, \ldots, N \quad \text{(10)}
\]

\[
F_{j+1}(q^{-1}) = q[F_j(q^{-1}) - E_jA_{s,j}(q^{-1})] \quad \text{(11)}
\]

\[
E_j = F_j(0) \quad \text{(12)}
\]

\[
E_0 = I \quad \text{(13)}
\]

Proof: Subtracting (7) from the equation obtained by substituting \( j \) with \( j+1 \) in (7), we get
In general, the variance of load demand between two sampling points can be seen to be very small, and therefore 

\[ A_{t+1}(q^{-1}) = A_{t}(q^{-1}). \]

In this way, (14) becomes

\[
[E_{t+1}(q^{-1}) - E_{t}(q^{-1})] A_{t+1}(q^{-1}) + q^{-1}[q^{-1}F_{t+1}(q^{-1}) - F_{t}(q^{-1})] = 0.
\]

From the above equation, we can see that the first \( j \)-term coefficients of \( E_{t+1}(q^{-1}) \) and \( E_{t}(q^{-1}) \) must be equal, so that (10)-(12) are established. If we let \( j = 1 \) in (7), the initial conditions (13) can be obtained. \( \square \)

**Theorem 3:** The solution of the Diophantine equation (8) may be obtained using the following recursion

\[
G_{t}(q^{-1}) = G_{t}(q^{-1}) + G_{j} q^{-j},
\]

\[
H_{t}(q^{-1}) = q[H_{t}(q^{-1}) + E_{t}B_{t}(q^{-1}) - T(q^{-1})G_{t}],
\]

\[
G_{j} = H_{j}(0) + E_{j}B_{j}(0),
\]

\[
G_{0} = E_{0}B_{0}(0), H_{1}(q^{-1}) = q[E_{0}B_{0}(q^{-1}) - T(q^{-1})G_{0}].
\]

The proof of theorem 3 is similar to the proof of theorem 2, and is omitted here.

### 4.2 The constrained multi-variable generalized predictive control (ExpMPC) strategy

Equations 5 and 6 may be used to obtain a vector-matrix form for the predicted output. We first define

\[
\tilde{Y}(t) = [Y(t+1)|t^{T}, Y(t+2)|t^{T}, \cdots, Y(t+N)|t^{T}]^{T}
\]

\[
\tilde{Y}_{0}(t) = [Y_{0}(t+1), Y_{0}(t+2), \cdots, Y_{0}(t+N)]^{T}
\]

\[
\tilde{U}(t) = [U(t)|t^{T}, U(t+1)|t^{T}, \cdots, U(t+N_{u}-1)|t^{T}]^{T}
\]

\[
\tilde{Y}_{0}(t) = [Y_{0}(t+1), Y_{0}(t+2), \cdots, Y_{0}(t+N)]^{T}
\]

here \( N \) is the prediction horizon whereas \( N_{u} \) is the control horizon after which controls are assumed to be zeros, i.e. \( U(t+j) = 0, j \geq N_{u} \). In the definitions above, \( \tilde{Y}_{0}(t) \) is the output reference sequence that usually may be designed as

\[
\begin{bmatrix}
Y_{i}(t), j = 0,1, \cdots, N-1 \\
Y_{i}(t + j + 1) = (1-\mu)Y_{i}(t + j) + \mu W^{i}, \mu \in (0,1)
\end{bmatrix}
\]

where

\[
W^{i} \text{ is the set-point. From (5-6), thus yields}
\]

\[
\tilde{Y}(t) = \tilde{G}_{i} \tilde{U}(t) + \tilde{Y}_{0}(t)
\]

Let the Exp-ARX model-based multi-step predictive control algorithm (ExpMPC) cost-function be defined as

\[
J = \left\| \tilde{Y}(t) - \tilde{Y}_{0}(t) \right\|_{R}^{2} + \left\| \tilde{Y}_{0}(t) \right\|_{R}^{2}
\]

where \( R = \text{diag}\{r_{1}, \cdots, r_{N_{u}}\} \) is the weighting matrix. Introducing (15) into (16), and ignoring constant terms, we obtain a quadratic version of the ExpMPC cost-function equivalent to (10)

\[
\tilde{J} = \frac{1}{2} \tilde{U}(t)^{T}(\tilde{G}_{i}^{T} \tilde{G}_{i} + R) \tilde{U}(t) + \left[ \tilde{Y}_{0}(t) - \tilde{Y}_{0}(t) \right]^{T} \tilde{G}_{i} \tilde{U}(t)
\]

The future optimal control sequence \( \tilde{U}(t) \) can be obtained by searching \( \tilde{U}(t) \) such that \( \tilde{J} \) is minimized under some constraints as follows

\[
\tilde{U}(t) = \arg \min_{\tilde{U}(t)} \tilde{J}
\]

subject to

\[
\begin{bmatrix}
\tilde{G}_{i} \\
-\tilde{G}_{i}
\end{bmatrix} \tilde{U}(t) \leq \begin{bmatrix}
\tilde{Y}_{0} - \tilde{Y}_{0} \\
\tilde{Y}_{0} + \tilde{Y}_{0}
\end{bmatrix}
\]

\[
\tilde{U}_{\text{min}} \leq \tilde{U}(t) \leq \tilde{U}_{\text{max}}
\]

In a actual power plant control, the first vector \( \tilde{U}(t) \) of the solution \( \tilde{U}(t) \) is used.
5. Simulation results
In the following simulation, $N = 20$, $N_u = 4$, $R = 35I$, $\mu = 0.4$, and the Exp-ARX model (2) identified in Section 3 are used as internal models of the ExpMPC strategy. The comparisons of the control performance of ExpMPC and the gain-scheduling PID controls are shown in Fig. 5. It is clear that over a wide load operating range, the control performance of ExpMPC based on the Exp-ARX models is far better than conventional PID control as shown in Fig. 5. It may also be seen from Fig. 6 that the ExpMPC based on the Exp-ARX models has demonstrably better control performance than a GPC using a global linear ARX model as its internal model. (Here, the global linear ARX model was identified using the same data and model orders as for the Exp-ARX model).

6. Conclusion
In this paper, a multivariable nonlinear exponential ARX (Exp-ARX) model has been presented, which can very closely describe the nonlinear dynamic behaviors of thermal power plants over the whole operating range. Since Exp-ARX models are similar to linear ARX models fixed at certain load levels or operating points, actually this is a class of local linearization models which does not need to resort to on-line parameter estimation. They may be applied to describe a class of nonlinear systems satisfying certain smooth conditions. On the other hand, the Exp-ARX model can be also regarded as a special NARMAX (nonlinear ARMAX) model. Notice that there is no necessity to give up the idea of the original NARMAX model as a result of the birth of the Exp-ARX model. However, in actual applications, using Exp-ARX models is very convenient, because some identification approaches of linear models may be used to identify the models.

Fig. 5 Control performance of ExpMPC and PID

Fig. 6 Control performance of ExpMPC and GPC based on a global linear ARX model
The MIMO constrained long-range predictive control strategy based on the Exp-ARX models proposed in this paper has shown much better control performance than conventional gain-scheduling PID control and the usual GPC based on a global linear ARX model. Since our suggested predictive controller does not resort to on-line parameter estimation, as is done in general self-tuning control, it also has better reliability than the usual self-adapting control algorithms.

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References
Appendix A. The plant model

1. Turbine/Main steam pressure system

\[
\frac{dx_1(t)}{dt} = -\frac{1}{T_f} [x_1(t) - u_2(t)]
\]

\[
\frac{dx_3(t)}{dt} = -\frac{1}{T_{FW}} [x_2(t) - K_{FWD}u_1(t)]
\]

\[
\frac{dx_5(t)}{dt} = -\frac{1}{T_{LP}} x_3(t) + \frac{K_{1P}u_0(t)u_2(t)}{T_{LP}f_2(u_0(t))} \left[ \frac{x_5(t)}{f_1(u_0(t))} + 1 \right] - x_4(t) + K_{FWD}K_{MT}(u_0(t))u_1(t) - x_6(t)
\]

\[
\frac{dx_7(t)}{dt} = 1 \left[ x_1(t) - x_2(t) + K_{FWD}u_1(t) \right] + u_0(t)u_1(t) \left[ \frac{x_5(t)}{T_B}f_2(u_0(t)) \right] - \frac{x_5(t)}{f_1(u_0(t))} - 1
\]

\[
\frac{dx_9(t)}{dt} = \frac{1}{T_{MT}(u_0(t))} \left[ K_{MT}(u_0(t))x_1(t) - K_{MT}(u_0(t))x_2(t) - x_6(t) + K_{FWD}K_{MT}(u_0(t))u_1(t) - \frac{K_{MT}(u_0(t))u_0(t)u_1(t)}{T_{MT}(u_0(t))f_2(u_0(t))} \left[ \frac{x_5(t)}{f_1(u_0(t))} + 1 \right] \right]
\]

2. Boiler/Reheater system

\[
\frac{dx_2(t)}{dt} = -\frac{1}{T_R(u_0(t))} \left[ x_1(t) + x_4(t) + x_7(t) - u_1(t) \right]
\]

\[
\frac{dx_4(t)}{dt} = -\frac{1}{T_{GD}(u_0(t))} \left[ x_6(t) + T_{RGD}(u_0(t))K_{GD}(u_0(t))u_5(t) - L_{RI} \right]
\]

\[
\frac{dx_6(t)}{dt} = -\frac{1}{T_{RHI}(u_0(t))} \left[ K_{RHI}x_3(t) + K_{RHI}x_4(t) + x_9(t) - x_{10}(t) - x_{11}(t) - x_{12}(t) - x_{13}(t) \right]
\]

\[
\frac{dx_8(t)}{dt} = \frac{2}{T_{RHI}(u_0(t))} \left[ x_{10}(t) + C_{RI}x_3(t) + C_{RI}x_4(t) - C_{RI}(u_2(t)) \right]
\]

\[
\frac{dx_10(t)}{dt} = \frac{2}{T_{RHI}(u_0(t))} \left[ K_{RHI}x_7(t) - x_8(t) - x_9(t) + x_{10}(t) - x_{11}(t) \right]
\]

\[
\frac{dx_11(t)}{dt} = \frac{2}{T_{RHI}(u_0(t))} \left[ K_{RHI}(u_2(t))x_{11}(t) - x_{12}(t) \right]
\]

\[
\frac{dx_12(t)}{dt} = \frac{2}{T_{RHI}(u_0(t))} \left[ K_{RHI}(u_2(t))x_{11}(t) - x_{12}(t) \right]
\]

\[
\frac{dx_13(t)}{dt} = \frac{K_{TG}}{T_{TG}(u_0(t))} \left[ x_3(t) + x_4(t) + \frac{1}{K_{TG}} x_{13}(t) - u_2(t) \right]
\]

\[
\frac{dx_{14}(t)}{dt} = -\frac{2}{T_{RV}(u_0(t))} \left[ K_{RHI}x_7(t) - x_{12}(t) - x_{13}(t) + x_{14}(t) \right]
\]

\[
\frac{dx_{15}(t)}{dt} = \frac{1}{T_{RHO}(u_0(t))} \left[ K_{RHO}(u_2(t))T_{RTE}(u_0(t))x_{14}(t) - x_{15}(t) \right]
\]

\[
\frac{dx_{16}(t)}{dt} = \frac{2}{T_{D}(u_0(t))} \left[ K_{RHO}(u_2(t))T_{RTE}(u_0(t))x_{14}(t) - x_{16}(t) \right]
\]

\[
\frac{dx_{17}(t)}{dt} = \frac{2}{T_{D}(u_0(t))} \left[ x_{16}(t) - x_{17}(t) \right]
\]

\[
\frac{dx_{18}(t)}{dt} = \frac{1}{T_{RHT}(u_0(t))} \left[ x_{17}(t) - x_{18}(t) \right]
\]

3. Boiler/Superheater system

\[
\frac{dx_{19}(t)}{dt} = -\frac{2}{T_{PR}(u_0(t))} \left[ x_{19}(t) + C_{SH}x_3(t) + C_{SH}x_4(t) - C_{SH}(u_2(t)) \right]
\]

\[
\frac{dx_{20}(t)}{dt} = \frac{1}{T_{s}(u_0(t))} \left[ x_3(t) + x_4(t) + x_{20}(t) - u_1(t) \right]
\]

\[
\frac{dx_{21}(t)}{dt} = \frac{1}{T_{SD}(u_0(t))} \left[ x_{21}(t) - K_{SD}(u_0(t))T_{SGD}(u_0(t))u_4(t) - L_{SH} \right]
\]

\[
\frac{dx_{22}(t)}{dt} = \frac{R_{Ri}}{T_{Ri}(u_0(t))} \left[ x_3(t) + x_4(t) + \frac{1}{R_{Ri}} x_{22}(t) - u_2(t) \right]
\]

\[
\frac{dx_{23}(t)}{dt} = \frac{2}{T_{PP}(u_0(t))} \left[ x_{19}(t) - K_{S}x_{20}(t) + x_{21}(t) + x_{22}(t) - x_{23}(t) \right]
\]

\[
\frac{dx_{24}(t)}{dt} = -\frac{1}{T_{S}(u_0(t))} \left[ x_{24}(t) - K_{S}(u_0(t))T_{S}(u_0(t))u_5(t) \right]
\]

\[
\frac{dx_{25}(t)}{dt} = \frac{2}{T_{PL}(u_0(t))} \left[ K_{PP}(u_2(t))x_{23}(t) + x_{24}(t) - x_{25}(t) - K_{S}(u_0(t))T_{S}(u_0(t))u_5(t) \right]
\]

\[
\frac{dx_{26}(t)}{dt} = \frac{K_{R}}{T_{R}(u_0(t))} \left[ x_3(t) + x_4(t) + \frac{1}{K_{R}} x_{26}(t) - u_2(t) \right]
\]

\[
\frac{dx_{27}(t)}{dt} = \frac{2}{T_{PL}(u_0(t))} \left[ K_{R}x_{20}(t) - x_{25}(t) - x_{26}(t) + x_{27}(t) \right]
\]

\[
\frac{dx_{28}(t)}{dt} = \frac{1}{T_{S}(u_0(t))} \left[ x_{26}(t) - K_{S}(u_0(t))T_{S}(u_0(t))u_5(t) \right]
\]

\[
\frac{dx_{29}(t)}{dt} = \frac{2}{T_{S}(u_0(t))} \left[ x_6(t) - K_{PP}(u_2(t))x_{27}(t) - x_{26}(t) + x_{29}(t) + K_{S}(u_0(t))T_{S}(u_0(t))u_6(t) \right]
\]
\[
\begin{align*}
\frac{dx_{30}(t)}{dt} &= -\frac{2}{T_{FS2}(u_0(t))} [K_2x_{20}(t) - x_{29}(t) + x_{30}(t) - x_{31}(t)] \\
\frac{dx_{31}(t)}{dt} &= -\frac{K_{RF}}{T_{RF}(u_0(t))} [x_3(t) + x_4(t) + \frac{1}{K_{RF}} x_{31}(t) - u_2(t)] \\
\frac{dx_{32}(t)}{dt} &= \frac{1}{T_{PLT}} [K_{PL}(u_2(t))x_{27}(t) - x_{32}(t)] \\
\frac{dx_{33}(t)}{dt} &= \frac{T_{STE}(u_0(t))}{T_{MSB}(u_0(t))} [K_{TP}x_5(t) + K_{SHO}(u_2(t))x_{30}(t)] \\
&\quad - \frac{1}{T_{MSB}} x_{33}(t) \\
\frac{dx_{34}(t)}{dt} &= \frac{2T_{STE}(u_0(t))}{T_{MS}(u_0(t))} [K_{TP}x_5(t) + K_{SHO}(u_2(t))x_{30}(t)] \\
&\quad - \frac{2}{T_{MS}(u_0(t))} x_{34}(t) \\
\frac{dx_{35}(t)}{dt} &= \frac{2}{T_{MS}(u_0(t))} [x_{35}(t) - x_{36}(t)] \\
\frac{dx_{36}(t)}{dt} &= \frac{2}{T_{MST}} [x_{35}(t) - x_{36}(t)] \\

4. \text{ PID controllers} \\
\frac{dx_{37}(t)}{dt} &= -\frac{K_{P0}(u_0(t))}{T_{I0}(u_0(t))} x_5(t) \\
\frac{dx_{38}(t)}{dt} &= -\frac{K_{MSB}K_{P1}(u_0(t))}{T_{I1}(u_0(t))} x_{33}(t) \\
\frac{dx_{39}(t)}{dt} &= -\frac{K_{P2}(u_0(t))}{T_{I2}(u_0(t))} [x_3(t) + x_4(t) - u_0(t)] \\
\frac{dx_{40}(t)}{dt} &= -\frac{K_{RHB}K_{P3}(u_0(t))}{T_{I3}(u_0(t))} x_{15}(t) \\

\text{SGD Demand:} \\
u_5(t) &= K_{RHB}K_{P3}(u_0(t))x_{15}(t) + x_{40}(t) + \text{RGDD}(t) \\
\text{RGD Demand:} \quad u_4(t) &= -u_5(t) \\
\text{SP1 Demand:} \quad u_3(t) &= f_{F1}(0.01x_{32}(t) + \text{SP1DD}(t)) \\
\text{SP2 Demand:} \quad u_6(t) &= f_{F2}(0.01K_{MSB}x_{33}(t) + \text{SP2DD}(t)) \\
\text{TCV Demand:} \quad u_7(t) &= f_{F3}[K_{P2}(u_0(t))(-x_5(t) - x_4(t) + u_0(t)) + x_{39}(t)] + f_2(u_6(t)) \\
\text{Turbine outlet steam flow:} \\
u_8(t) &= \frac{u_0(t)u_2(t)}{f_2(u_6(t))} [\frac{x_5(t)}{f_1(u_0(t))} + 1].
\end{align*}
\]

Notice that the inputs defined above are the input signals in the simulation model, and are different to the outputs used by the predictive controller proposed in this paper.

\[C_{SH}(): \text{ primary superheater inlet steam temperature;} \]
\[C_{RFF}(): \text{ reheater inlet steam temperature;} \]
\[f_1(): \text{ relationship between MWD and TP set-point;} \]
\[f_2(): \text{ relationship between MWD and TCV set-point.} \]

\text{Limiters:}
\[f_{F1}(x) = \begin{cases} 
-2, & x < -2 \\
x, & -2 < x < 4 \\
4, & x > 4 
\end{cases} \]
\[f_{F2}(x) = \begin{cases} 
-4, & x < -4 \\
x, & -4 < x < 6 \\
6, & x > 6 
\end{cases} \]
\[f_{F3}(x) = \begin{cases} 
-10, & x < -10 \\
x, & -10 < x < 10 \\
10, & x > 10 
\end{cases} \]