Monitoring the stability of BWR oscillation by nonlinear time series modeling

Zhaoyun Shi *, Yoshiyasu Tamura, Tohru Ozaki

The Institute of Statistical Mathematics, 4-6-7 Minami-Azabu, Minato-ku, Tokyo 106-8569, Japan

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Abstract

Monitoring the dynamics evolution of BWR oscillation has great importance in evaluating safety of the nuclear systems. Time series analysis methodology has been widely accepted as a powerful tool for this subject. BWR stability has been so far evaluated by decaying ratio (DR) calculated from the impulse response function of autoregressive (AR) model. To explore much more reliable method for detecting BWR instability, this paper introduces a nonlinear time series analysis approach namely exponential autoregressive (ExpAR) modeling. The ExpAR model is available for revealing types of nonlinear dynamics such as fixed point, limit cycle, and even chaos. Furthermore, the model is real-time estimated so that it is suitable for the purpose of on-line BWR instability detection. Empirical analysis of typical benchmark neutronic signal shows the effectiveness of this proposal. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Monitoring stability of boiling water reactor (BWR) is one of attractive subjects in nuclear energy field. Since it is relevant to evaluating the safety of a nuclear system, efforts both from theoretical and practical viewpoints have been made to correctly detect the early instability of BWR during last decades. In the literature, time series methodology has been accepted as one of powerful tools for this subject. The potential application of time series analysis is in determining some stability parameters (we here would rather to say indices) from underlying neutronic signals. In practice, one of the main stability indices widely accepted is decay ratio (DR).

* Corresponding author. Tel.: +81-3-3446-1501; fax: +81-3-5421-8796.
E-mail address: zyshi@ism.ac.jp (Z. Shi).

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Intuitively, the DR of underlying signal should approach near unity whenever limit cycle type oscillation occurs. However, since the common way to calculate DR is by linear autoregressive (AR) modeling of the neutronic signals, the value of DR is usually less than one except that very strong limit cycle oscillation has been occurred or oscillation is explosive. Thus, we can understand that, from the viewpoint of early detection of instability, US Nuclear Regulatory Commission safeguard regulations require that a BWR should not operate in regimes whenever the decay ratio would become larger than 0.8. As indicated by Analystis and Karlsson (1999), this would guarantee that the reactor power would not exceed the nominal power for an unacceptably long period of time with the undesirable consequences that the critical heat flux could locally be exceeded.

On the other hand, a recent study by Van der Hagen et al. (1997) indicated that the decay ratio is not a good index for the stability measure based on their experimental analysis for the Dodewaard natural circulation boiling water reactor. They found an interesting phenomenon that the decay ratio may change largely with a small change of operating conditions. They illustrated by experimental examples that the decay ratio is an unpractical and unsafe stability parameter for values above 0.6 since the reactor stability appears to be very sensitive to operational conditions for decay ratio above 0.6. We think it is a reasonable indication because the decay ratio they used is calculated from the impulse response of linear AR modeling (Van der Hagen et al., 1990), which is not necessarily an intrinsic index of limit cycle, and is only suitable for characterizing the linear damping oscillation. Thus it remains an open problem to explore some intrinsic features relevant to the instability of BWR oscillation.

Now that we have regarded limit cycle type oscillation of neutronic signal as the symbol of stability collapse for BWR operator, a straightforward idea is to detect the instability by using the intrinsic feature of limit cycle. The occurrence mechanism of limit cycle oscillation is too complex to clarify, it can be due to self-excitation or any stochastic forcing excitation. However, the appearance of any limit cycle oscillations seems definite both in time and frequency domains. In time domain, oscillation is aggravated by limit cycle with appearance that the amplitude gradually becomes larger and larger towards an unacceptably high level. In frequency aspect, limit cycle may lead to the appearance of multiple peaks in power spectral of oscillating signal, say higher harmonics phenomenon. Higher harmonics phenomenon in neutronic spectra has been observed in regional oscillation of BWR (Van der Hagen et al., 1997; Analystis and Karlsson, 1999). Therefore, Analystis and Karlsson (1999) proposed that the nonlinear effects such as higher harmonics in neutronic spectra could be present in stability measurements at commercial BWRs. The idea on using nonlinear characteristics of limit cycle oscillation as indices for monitoring purpose is interesting. However, as Analystis and Karlsson (1999) themselves indicated, due to the simultaneous presence of background noise in oscillation signal, the higher harmonics in the neutron power spectral can not be easily seen, for which we will show evidence by our aforementioned exponential autoregressive (ExpAR) model. In this sense, in neutronic spectra, higher harmonics usually appears only when the amplitude of neutronic oscillation signal becomes large, in which case the effects of noise become trivial. In other words, only violent oscillation of limit cycle
type will result in clearly visible multiple peaks in spectra. This gives an implication that higher harmonics seems not a good instability measurement of BWR oscillation from the viewpoint of early detection of fault due to the existence of stochastic noise.

In this paper, we provide a new approach to detecting limit cycle behavior from BWR oscillating signal by nonlinear time series modeling. The nonlinear time series model that we use in this study is namely exponential autoregressive (ExpAR) model, which is provided by Ozaki and Oda (1978) when they analyzed ship-rolling process as a typically complicated nonlinear vibration. The ExpAR model has been well-known being able to reveal complex nonlinear dynamics such as singular point, limit cycle, and even chaos. It has been accepted as one of classic model in nonlinear time series analysis field (Priestley, 1988). Several successful applications have been achieved mainly in nonlinear stochastic vibration analysis (Haggan and Ozaki, 1981, Shi et al., 1998). In Section 2, we will first introduce the ExpAR model with respect to its nonlinear dynamics representability, and then give a real-time estimate method. Section 3 presents data analysis results of measured average power range monitor (APRM) and local PRM (LPRM) data that have been used for stability Benchmark (Verdú et al., 1998). The results show that the approach to BWR stability detection by the ExpAR modeling is very reliable and of robustness. In final section we give concluding remarks.

2. Nonlinear time series models and dynamics detection

As we all know, the study of nonlinear science started from the motivation to understand complex oscillations that are absentlly interpretable by linear oscillation theory. One of the first emerged concepts in nonlinear oscillation theory was limit cycle largely as a result of the work of Poincaré. A number of differential equations including Van der Pol equation and Duffing equation have been so far provided to analyze nonlinear oscillation, however, problems remain that, in most cases, especially for some complex dynamic systems, the differential equation approach is difficult to find analytical solutions to describe phenomena of interest.

About 20 years ago, some time series analysts considered this problem of understanding dynamic systems from viewpoint of statistics as well as traditional dynamic systems theory. The focus of this approach is on modeling complex systems from experimental data with difference equations. This approach has current popularity and is even accepted by traditional dynamic system analysts (Rapp et al., 1999), achievements of the approach have been establishing the theory of this still developing subject of nonlinear time series analysis. In this study, we are particularly interested in some nonlinear time series models developed in the early stage for their tight relevance to nonlinear oscillation analysis. One of this type of models can be the ExpAR model (Ozaki, 1985). We introduce the ExpAR model to stability monitoring problem of BWR oscillation for its ease in both physical interpretation and practical use, which will be discussed in following subsections.
2.1. ExpAR model and nonlinear properties

Assume that a time series \( \{x(t)\} \) with its constant mean removed is represented by a simple ExpAR model written as follows,

\[
x(t) = \sum_{i=1}^{p} (\phi_i + \pi_i \exp(-\gamma x^2(t-1)))x(t-i) + e(t)
\]

(1)

where \( \{e(t)\} \) is a sequence of Gaussian white noise, \( p \) denotes order of the model, \( \{\phi_i, \pi_i, i = 1, \ldots, p\} \) are model parameters, and \( \gamma \) is a dull parameter which is related to the scaling parameter of Gaussian distribution function, and it can be regarded as an adjusting parameter of the amplitude of underlying time series. In other words, any change of the value \( \gamma \) will not result in any change of nonlinear dynamics that the model possesses. This emphasis is important especially when we investigate the simulation of ExpAR model and consider the identification of this model.

The ExpAR model [Eq. (1)] is of AR model structure but the amplitude-dependent functional coefficients, this allows us to interpret how the dynamics of the ExpAR model follows dynamics evolution of the underlying process by the corresponding instantaneous characteristic equation of the model,

\[
\lambda^p - (\phi_1 + \pi_1 \exp(-\gamma x^2(t-1)))\lambda^{p-1} - \cdots - (\phi_p + \pi_p \exp(-\gamma x^2(t-1))) = 0
\]

(2)

where \( \lambda(t) \) denotes the instantaneous characteristic root of the ExpAR model [Eq. (1)]. Due to the continuous and bounded properties of the Gaussian function, the characteristic roots of the ExpAR model will change within two extremes. That is, when the amplitude \( \lvert x(t-1) \rvert \) tends to be enough large, the term \( \exp(-\gamma x^2(t-1)) \) will become zero, then the characteristic equation (Eq. (2)) has the following extreme form;

\[
\lambda^p - \phi_1 \lambda^{p-1} - \cdots - \phi_p = 0
\]

(3)

On the other hand, whenever \( \lvert x(t-1) \rvert \rightarrow 0, \exp(-\gamma x^2(t-1)) \) will become one, then the characteristic equation (Eq. (2)) of the model has following extreme form.

\[
\lambda^p - (\phi_1 + \pi_1)\lambda^{p-1} - \cdots - (\phi_p + \pi_p) = 0
\]

(4)

Thus, the two extreme equations (Eqs. (3) and (4)), or say the parameters \( \{\phi_i, \pi_i\} \), decide the dynamics evolution of the ExpAR model. Practically, in even very simple ExpAR models, just by adjusting the parameters \( \{\phi_i, \pi_i\} \), we can observe some typical nonlinear phenomena such as singular point, limit cycle, and period doubling bifurcation to chaos (Shi et al., 1999). In this mean, we can expect the ExpAR model to exhibit rich nonlinear dynamics by controlling the parameters of the model. Moreover, with respect to the problem being considered in this study, to detect limit
cycle behavior in oscillation signal, the convenience is that we have known the conditions for the existence of limit cycle behavior in the ExpAR model (Haggen and Ozaki, 1981), which are summarized as follows:

1. All the roots of the characteristic equation (Eq. (3)) lie inside the unit circle.
2. Some roots of the characteristic equation (Eq. (4)) lie outside the unit circle.

The condition (1) guarantees the process to be stable, it make the next state not to diverge too far away whenever the current amplitude is sufficiently large. The condition (2) makes the ExpAR model to have energy allowing the next state to diverge away from origin. Therefore, if the above two conditions are satisfied, the interplay of these two effects of opposite tendency produces a steady oscillation of certain amplitude, which may be of limit cycle type. The frequency of the oscillation is dependent on the power of the attracting and repelling energy that can be shown by the places of the extreme roots round unit circle. Note that the above two conditions are necessary to produce a limit cycle but not sufficient. A sufficient condition for the existence of a limit cycle is

$$\left(1 - \sum_{i=1}^{P} \varphi_i \right)/\sum_{i=1}^{P} \pi_i > 1 \text{ or } < 0 \quad \text{(5)}$$

The sufficient condition (3) is to guarantee that a singular point (fixed point) does not exist for the ExpAR model. However, sometimes an ExpAR model unsatisfying condition (3) may still have limit cycle. Ozaki (1982) indicated that this is because the singular points themselves of the model are unstable. So we have to use a supplementary condition (A1) to check whether the singular points are stable or not whenever the condition (3) is unsatisfied.

A1. The singular point of ExpAR model [Eq. (1)], if it exists, is stable if and only if the roots of the equation $\lambda^p - \beta_1 \lambda^{p-1} - \cdots - \beta_p = 0$ lie inside the unit circle, where $\beta_i$ are given as

$$\beta_1 = (\pi_1 + \varphi_1 \sum_{j=1}^{P} \pi_j - \pi_1 \sum_{j=1}^{P} \varphi_j)/\sum_{j=1}^{P} \pi_j + 2(1 - \sum_{j=1}^{P} \varphi_j)\log\left(1 - \sum_{j=1}^{P} \varphi_j/\sum_{j=1}^{P} \pi_j\right)$$

$$\beta_i = (\pi_i + \varphi_i \sum_{j=1}^{P} \pi_j - \pi_i \sum_{j=1}^{P} \varphi_j)/\sum_{j=1}^{P} \pi_j \quad (i = 2, 3, \cdots, p) \quad \text{(6)}$$

In order to illustrate the ability of the ExpAR model to exhibit limit cycle, we give simulations of a simple ExpAR model that is satisfactory with the above three conditions, which are shown in Fig. 1. Fig. 1(a) is the simulation of the deterministic part (namely skeleton in Statistics) of a second-order ExpAR model, which shows the true dynamics of the model. Fig. 1(b) shows the simulation of the stochastic
ExpAR model, which acts as a random oscillation process in which the dynamics is not visually clear. Furthermore, we also calculated the power spectral of the simulated data sets, which are shown in Fig. 2. Clearly, the multiple resonance peaks are visible in Fig. 2(a), this means that the higher harmonics phenomenon is exactly observable in the spectra of limit cycle type oscillation signal. Unfortunately, as shown in Fig. 2(b), when the oscillation signal is contaminated by noise, the higher harmonics in the spectra of the noisy signal becomes invisible. This example implies that the detection of higher harmonics in spectral is strongly dependent on the ratio of signal to noise.

After all, for a given time series, by building its ExpAR model, we can extract information about the existence of limit cycle behavior through checking the estimated model of the time series on above conditions. This can be a useful tool for detecting limit cycle behavior in BWR oscillation signal. Just by comparing absolute values of the roots of two extreme characteristic equations with unity and other simple calculation, we can make a definite decision “yes” or “no” on the limit cycle behavior of underlying signal.

2.2. On decay ratio of oscillation

As indicated by Van der Hagen et al. (1997), the stability of BWR oscillation has been conventionally expressed in decay ratio that is defined as the ratio between
consecutive maxima of the impulse response. For most cases, the impulse response is obtained by linear autoregressive (AR) model written as follows,

$$x(t) = \alpha_1 x(t-1) + \cdots + \alpha_p x(t-p) + e(t)$$  \hspace{1cm} (7)

where $e(t)$ denotes the white noise. The impulse response $h(t)$ is defined as follows:

$$h(0) = 1.0$$

$$h(t) = h(0) + \sum_{i=1}^{p} \alpha_i h(t-i)$$  \hspace{1cm} (8)

The impulse response function contains information about the process dynamics, a plot of impulse response visually gives us impression on the overall system’s dynamic response. We can understand the conditions on system stability by looking at the decaying speed of the response, which can be also interpreted by the places of characteristic roots of the dynamic model round unit circle. For the linear AR model fitted to a stationary time series, all of its roots should be inside the unit circle, therefore the curve of impulse response always tends to be decaying with oscillation. It is reasonable that the decay ratio $DR$ of an impulse response is approximately calculated as

$$DR = E[(P_{n+1} - P_n)/(P_n - P_{n-1})^2]$$  \hspace{1cm} (9)
where $E_{\|}$ means the ensemble average and $P_n$ is the $n$th peak amplitude of the impulse response function. By this definition, it is imaginable that the decay ratio DR should be always less than unity for a stable oscillation data set. However, for an unstable series which usually acts as oscillation of gradually diverging amplitude, if the AR model is estimated by some methods like the least squares and Burg methods except for Yule-Walker estimator, the parameters can be adaptively estimated so that some roots may lie in or outside the unit circle, details on parameter estimation of autoregressive model with respect to nuclear reactor analysis can be referred to De Hoon et al. (1996). Due to the existence of those characteristic roots of their absolute values equal or larger than unity, the peaks of the impulse response will gradually become larger and larger. In those cases, it is obviously seen that the decay ratio DR will be unity or of larger value than unity, which has also been observed by Van der Hagen et al. (1997). Thus, the approach to detecting instability of BWR oscillation through monitoring the decay ratio calculated from linear AR model sounds possible. However, the remained difficulty in this approach is how to select the threshold of the decay ratio to decision making of stability.

The impulse response (or say Green function in stochastic process theory) has been historically used to interpret system stability just as the same as that autocovariance function plays a major role in statistical time series analysis. However, for nonlinear systems analysis, another way like simulation of a nonlinear model takes the same role in exploring systems dynamics. Simulation means a reproduced series from a model with initial values. By looking at the simulation, we can know visually whether the system is stable (damping), asymptotic stable (limit cycle) or unstable (explosive), certainly the assumption is that the model should be a correct description of the underlying system. For oscillation data analysis, we can even calculate an alternative decay ratio from a simulation plot with the same idea as Eq. (9). Then a decision can be easily made on the stability of the oscillation systems from the simulation pattern as follows: if the decay ratio is one, then system is of limit cycle behavior; if decay ratio is larger than one, it is seriously unstable; if decay ratio is less than one, system is stable. Thus the threshold of so-called alternative decaying ratio is definitely one. We will show the simulations of the estimated ExpAR model of BWR oscillation signal with different developing stages of stability in the section of numerical examples.

2.3. Real-time estimate of the ExpAR model

For the problem of on-line monitoring a dynamic system, a common way is to segment on-line measured signal of a running system. Then it becomes very important to achieve a fast decision-making through analyzing each segment of measured signal. In this study, real-time identification of the ExpAR model becomes a crucial task to achieve successful application of the ExpAR model. For the exponential autoregressive model [Eq. (1)] representing a zero-mean time series $\{x(t), t = 1, \ldots, N\}$, the unknown parameters to be estimated are a set of linear parameters $\theta = \{\varphi_i, \pi_i, i = 1, \ldots, p\}$ and one nonlinear parameter $\gamma$. Thus, what we have to do for the ExpAR model estimation is to estimate the white noise variance
$\sigma^2_e$ and the system parameters $\theta = \{\gamma, (\phi_i, \pi_i, i = 1, \ldots, p)\}$. Having an ExpAR model, the time series $\{x(t), t = 1, \ldots, N\}$ is transformed back to noise input series $\{e(t), t = 1, \ldots, N\}$ with variance $\sigma^2_e$.

$$e(t) = x(t) - \sum_{i=1}^{p}(\phi_i + \pi_i \exp(-\gamma x^2(t-1)))x(t-i)$$

We can apply maximum likelihood method to estimate the unknown parameters. The Gaussian log-likelihood function $L(\theta)$ of the data is shown as the follows,

$$L(\theta) = \log f(x(1), x(2), \ldots, x(N); \theta) = \sum_{t=1}^{N} \log f(x(t)|x(t-1); \theta)$$

$$= -\frac{N}{2} \log(2\pi \sigma^2_e) - \frac{1}{2\sigma^2_e} \sum_{t=1}^{N} e^2(t)$$

When we calculate the noise variance $\sigma^2_e$ by maximizing the log-likelihood $L(\theta)$, we have the relation $\hat{\sigma}^2_e = \sigma^2_e$

$$\frac{\partial}{\partial \sigma^2_e} L(\theta) = -\frac{N}{2\sigma^2_e} + \frac{1}{2\sigma^2_e} \sum_{t=1}^{N} e^2(t) = 0$$

at the maximum likelihood point. Thus the maximum likelihood estimate $\hat{\theta}$ is obtained by minimizing the variance of the prediction errors, say the least squares method,

$$J(\theta) = \frac{1}{N} \sum_{t=1}^{N} \left( x(t) - \sum_{i=1}^{p}(\phi_i + \pi_i \exp(-\gamma x^2(t-1)))x(t-i) \right)^2$$

This is exactly a nonlinear optimization procedure that is commonly time consuming due to the existence of the nonlinear parameter $\gamma$, for which we refer to some estimation procedures (Haggan and Ozaki, 1981; Shi and Aoyama, 1997). Note that the scaling parameter $\gamma$ in the ExpAR model is a rather dull parameter. If we can assign specific value for $\gamma$ beforehand, then parameter estimate of the ExpAR model reduces to the problem of linear regression estimate. It can be implemented simply by solving a linear normal equation.

In terms of the mechanism of the ExpAR model to reveal the limit cycle, it can be seen that the $\gamma$ value as a scaling parameter takes the role in adjusting the instantaneous roots of the model. Whenever the state $x(t-1)$ becomes far away from the equilibrium point, $(\phi_i + \pi_i \exp(-\gamma x^2(t-1)))$ terms in the ExpAR model should become $\phi_i$, in other words, the nonlinear term $\exp(-\gamma x^2(t-1))$ should be zero, so that the resulting model has all roots less than unit to force the next state $x(t)$ not to
diverge further. From this viewpoint, we think that the nonlinear coefficient \( \gamma \) can be determined heuristically from the original data set, and define

\[
\gamma = - \frac{\log e}{\max_{t-1} \{ x^2(t-1) \}}
\]  

(12)

where \( e \) is a small number, and \( \max_{t-1} \{ x^2(t-1) \} \) represents the square of the maximum amplitude among the data set. With this definition, the model coefficients can insist on changing toward constants \( \varphi_i \) even if the observation moves far away from the equilibrium, since \( \exp(-\gamma \max_{t-1} \{ x^2(t-1) \}) = e \), i.e., approximately zero. On the other hand, although this definition of \( \gamma \), the model coefficients \( \{ \varphi_i + \pi_i \exp(-\gamma x^2(t-1)) \} \) can always become \( \varphi_i + \pi_i \) when the state \( x(t-1) \) moves to zero. So the instantaneous model may have some roots outside the unit circle to force the next state to increase. Thus, based on this definition of \( \gamma \) value, though it is not optimal, the ExpAR model is still assured to reveal the limit cycle behavior of the time series underlying.

Beneficial from this idea, the resulting convenience is that the \( \gamma \) value achieves to be quickly determined from original data set. Accordingly, real-time estimation of the ExpAR model becomes easily achieved because the estimate of other linear coefficients by the least squares [Eq. (11)] can be very quickly implemented for a modern personal computer. For the estimate of another parameter, the selection of order \( p \), we use Akaike information criterion (AIC) (Akaike, 1974).

\[
AIC = 2(-L(\theta) + N_\theta)
\]  

(13)

Here, \( L(\theta) \) is the log-likelihood, \( N_\theta \) denotes the number of unknown parameter in model, for AR model and ExpAR model of order \( p \), \( N_\theta \) becomes \( p \) and \( 2p + 1 \), respectively.

3. Numerical examples

The neutronic oscillation signal we used here to illustration is the data given in Forsmarks 1 and 2 stability benchmark, for details, readers can refer to the introduction of that benchmark (Verdú et al., 1998). One of the objectives of that benchmark, as announced in that introduction, is the comparison of different time series analysis methods that can be applied to the study of BWR stability. According to (Verdú et al., 1998), the benchmark database is divided into six cases, the sampling rate of all the time series being 25 Hz, decimated to 12.5 Hz. We here show the analyzing results of data sets collected in case 4. The very short introduction tells us that the case 4 contains a mixture between a global oscillation mode and a regional (half core) oscillation. The case consists of APRM and LPRM (local PRM) signals coming from one measurement, the introduction gives showing of the LPRM positions in the core, and other description of the LPRM signals.
Our interest to these signals is to make clear what stability stages the oscillation signals are belonged to, are they stable, critical stable or unstable? In turn, through the analysis, we want to appeal the effectiveness of our proposal. Each data set included in case 4 has 4209 observations; we use the first 1023 samples for analyzing purposes. For comparison, we use two approaches to empirically analyzing, one is to calculate the decay ratio DR through linear autoregressive modeling as a traditional way, another approach is to extract the limit cycle behavior through the nonlinear exponential autoregressive modeling. The analyzing results from the two approaches are shown in Table 1, which include information like the model orders selected by minimized AIC criterion, AIC values, residual variances of two models. The table also gives the DR values by AR modeling and the limit cycle behavior extracted by ExpAR modeling, in which “Yes” denotes the existence of limit cycle behavior in estimated ExpAR models; “No” denotes that there is no limit cycle behavior. The interesting is that, for the time series that has DR value exceeding 0.8 from AR modeling approach, the ExpAR modeling approach does extract the limit cycle behavior embedded in the data. This proves the reasonability for US Nuclear Regulatory Commission to select 0.8 as a threshold of DR to warn the collapse of BWR stability. On the other hand, for neutron oscillation signals measured in some positions, even though their DR values are smaller than 0.8, their ExpAR

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</tbody>
</table>
Fig. 3. Original (mean removed) neutronic signal (upper) and simulation of the estimated ExpAR models (bottom). The initial values $x(1), \ldots, x(p)$ are all arbitrarily selected to be one in all three simulations. The decaying ratio (DR) calculated by linear AR modeling of the three data sets are as follows: (a) DR = 0.8155; (b) DR = 0.5998; (c) DR = 0.2395. It is obviously seen that alternative DR values calculated from the simulation of ExpAR models can be expected to be unity for series a and b, but very less than one for series c.
models still extract limit cycle behavior. This fact supports the discovery of Van der Hagen et al. (1997) that the decay ratio is not a robust index for the stability measure. In order to visually check the limit cycle behavior, we demonstrate the simulation of the estimated ExpAR models, which are shown in Fig. 3. Fig. 3 includes three typical sets of BWR oscillation signal (C4_lprm.1, C4_lprm.7, and C4_lprm.8 in case 4) with respect to different levels of DR value. As expected, the simulations of ExpAR models of the first two data sets act as limit cycle type, while the last simulation acts as fixed point type. The simulations agree with the numerical results shown in Table 1. Thus we have two routines to detect limit cycle if the ExpAR modeling approach is applied, one is to numerically check the conditions [Eqs. (3)–(6)] on limit cycle existence of the estimated model, another is to visually check simulation of the estimated model. All these illustrations propose that the ExpAR modeling approach is applicable for monitoring the stability of oscillation signal.

4. Conclusion

We introduced nonlinear exponential autoregressive modeling for the implementation of BWR oscillation monitoring. The ability to detect limit cycle type oscillation of this approach has been shown by benchmark data analysis. We give emphasis on using nonlinear time series analysis methodologies to BWR oscillation monitoring due to the fact that the limit cycle, as a crucial feature characterizing BWR stability, is a nonlinear concept. Linear autoregressive modeling is possibly available for extracting the feature in a sense that limit cycle signal usually appears to be of periodic behavior. Note that the autocorrelation function or impulse response function of the estimated AR model of a cyclical series usually acts damping cyclical evolution. However, it is not enough robust to make decision that a cyclical series is of limit cycle behavior only by looking at the decay ratio of response function of the linear model. Conversely, we would rather to believe the truth of nonlinear information extracted from a nonlinear model. In terms of the ExpAR model’s reasonable explanation of limit cycle vibration, we think it is more suitable for BWR stability monitoring than AR modeling approach, although the ExpAR models detects limit cycle in only cases where DR < 1 in this benchmark study. It is our great pleasure that one can check the robustness of our approach using different data. At least, we think the ExpAR model approach is of much safer to application than AR modeling since the former detect limit cycle in cases the latter says no, because of the particular necessity of a safe system in nuclear industry.

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References