A Study on Real-time Detecting of Machine Tool Chatter

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A Study on Real-time Detecting of Machine Tool Chatter*

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A unified nonlinear time series analysis approach to the problem of feature extraction of machine tool chatter is developed. Considering the mechanism of chatter development, the procedure proposes the limit cycle behavior of self-excited random vibration to be the intrinsic index of chatter occurrence, and provides the exponential autoregressive (EAR) model to extract the index from on-line measured machining signal. In order to real-time implement this procedure, a quickly estimating method of the EAR model is also provided by introducing a heuristic determination of the nonlinear coefficient in the model. Finally both the simulation and experimental data are used to shown the effectiveness of the procedure.

Key words: machine tool chatter, limit cycle, feature extraction, real-time

1. Introduction

Machine tool chatter as a nuisance in machining process is highly undesirable for its detrimental effect on surface finish, dimensional accuracy, tool life as well as machine life. From a process automation point of view, it is therefore necessary that an intelligent monitoring system [1] be devised to detect the progress of machine tool chatter during machining operations so that chatter can be identified and in turn controlled. The purpose of this study is to develop such a reliable and cost effective method for monitoring machine tool chatter real-time by using time series analysis techniques.

In terms of the fact that the machine tool chatter is a nonlinear oscillation of the limit cycle type [2], which can be regarded as an intrinsic property independent of process working conditions and measuring noise, Shi and Aoyama [3] proposed the limit cycle behavior of cutting vibration signal to be the index of chatter occurrence, and provided to employ the nonlinear time series exponential autoregressive (EAR) model [4] to achieve the extraction of the limit cycle feature. Although the effectiveness of the proposal, as shown therein, it is still expected to be real-time implemented for actual applications, in which the on-line estimation of the EAR model is the key task. This paper first explains how the EAR model describes the limit cycle behavior, and then gives a method of real-time estimation of the EAR model by introducing the heuristic determination of the nonlinear coefficient in the model. Finally both the simulation and experimental data analysis are illustrated to show the effectiveness.

2. Feature extraction by the EAR model

Since the limit cycle is the intrinsic feature of machine tool chatter, it is reliable to use it as the index of chatter occurrence. However the task remains how to detect this index from the observed vibration signal of machining process and in turn make such a numerical decision that chatter happened.

Limit cycle can not be found directly from a time series except for plotting correlation graphs, but flexible automatic manufacturing systems are fond of any numerical information like "yes" or "no" that is more precise and reliable to make decision for a computer. Therefore one has to develop methods extracting information about limit cycle by modeling. Since the self-excited vibration is really periodic oscillation of limited amplitude, the model available for representing chatter signal \( \{ x_t \} \) should have following necessary abilities such as: when \( |x_{t-1}| \rightarrow 0 \), the characteristic roots of the model must
be out of the unit circle in order to protect the \( x_t \) not to decay to zero; when \( |x_{t-1}| \to \infty \), the roots must be inside of unit circle to avoid the amplitude of \( x_t \) not to diverge further.

Fortunately, the EAR model has such properties that it can be an efficient tool for the numerical judgment of signal's limit cycle behavior. It is of simpler model structure similar to autoregressive (AR) model except for the state-dependent coefficients, usually the model is written as

\[
x_t = \sum_{i=1}^{p} (\varphi_i + \pi_i \exp(-\gamma x_{t-1}^2)) x_{t-i} + e_t
\]

where the nonlinear coefficient \( \gamma \) is a scaling parameter, \( \{\varphi_i, \pi_i\} \) are linear weight coefficients, \( p \) is the model order, and \( \{e_t\} \) is Gaussian white noise. Haggan and Ozaki\(^5\) had given the conditions for the existence of limit cycle behavior in the model:

[a] All the roots of the characteristic equation:

\[
\lambda^p - \varphi_1 \lambda^{p-1} - \varphi_2 \lambda^{p-2} - \cdots - \varphi_p = 0
\]

lie inside the unit circle. Therefore \( x_t \) starts to damp out when \( |x_{t-1}| \) becomes too large, while if the model satisfies the condition [b] such that

[b] Some roots of the characteristic equation:

\[
\lambda^p - (\varphi_1 + \pi_1) \lambda^{p-1} - (\varphi_2 + \pi_2) \lambda^{p-2} - \cdots - (\varphi_p + \pi_p) = 0
\]

lie outside the unit circle, then \( x_t \) starts to oscillate and diverge for small \( |x_{t-1}| \). The result of these two effects is expected to produce a similar sort of self-excited oscillation. Note that the above two conditions are necessary to produce a limit cycle but not sufficient. A sufficient condition for the existence of a limit cycle is

[c] \( (1 - \sum_{i=1}^{p} \varphi_i) / \sum_{i=1}^{p} \pi_i > 1 \) or \( < 0 \)

The objective of the sufficient condition [c] is to guarantee that a singular point (or fixed point) does not exist for the EAR model. However, some EAR models without satisfying condition [c] still have limit cycle. Ozaki\(^6\) indicates that this is because the singular points of the model are unstable. He gives a supplementary condition [c1] to check whether the singular points are stable or not whenever the condition [c] has not been satisfied.

[c1] The singular point of EAR model (1), if it exist, is stable if and only if the roots of the equation

\[
\lambda^p - h_1 \lambda^{p-1} - \cdots - h_p = 0
\]

lie inside the unit circle, where \( h_i \) are given as

\[
h_i = (\pi_i + \varphi_i \sum_{j=1}^{p} \varphi_j - \pi_i \sum_{j=1}^{p} \pi_j) / \sum_{j=1}^{p} \pi_j
\]

Thus, for a given time series, by building its EAR model, one can know the information of limit cycle existence through checking the estimated model of the series about above conditions.

3. Real-time estimate of the EAR model

The estimate of the EAR model, which is really a nonlinear optimization problem due to the nonlinear coefficient \( \gamma \) in the exponential term, has been considered by Haggan and Ozaki\(^5\), Shi and Aoyama\(^7\). However, for the objective of application in manufacturing system, the model has still waited for being estimated real-time. From the equation (1) it is obviously seen that the most important task in real-time estimating of the EAR model is how to determine the nonlinear coefficient \( \gamma \) quickly, since the estimate of other linear coefficients \( \{\varphi_i, \pi_i\} \) in the model is only a linear least squares problem whenever the \( \gamma \) is determined.

In terms of the mechanism of the EAR model to reveal the limit cycle or cyclical behavior, it can be seen that the \( \gamma \) as the scaling parameter takes the role in adjusting the instantaneous roots of the model. Whenever the state \( x_{t-1} \) becomes far away from the equilibrium point, \( \{\varphi_i, \pi_i \exp(-\gamma x_{t-1}^2)\} \) terms in the EAR model should be \( \varphi_i \), in other words, the nonlinear term \( \exp(-\gamma x_{t-1}^2) \) should approach zero, so that the
resulting model has all roots less than unit to force the next state \( x_t \) not to diverge further. From this viewpoint, we think that the nonlinear coefficient \( \gamma \) can be determined heuristically from the original data set, and define

\[
\gamma = -\log \frac{\varepsilon}{\max_{i \in I} \{x_0^2\}}
\]

(2)

where \( \varepsilon \) is a small number, and \( \max_{i \in I} \{x_0^2\} \) represents the square of the maximum amplitude among the data sets. With this definition, the model coefficients can insist on approaching constants \( \phi_i \), even if the observation moves far away from the equilibrium, since \( \exp(-\gamma \max_{i \in I} \{x_0^2\}) = \varepsilon \) i.e. approximately zero; on the other hand, although this definition of \( \gamma \), the model coefficients \( \{\phi_i + \pi_i \exp(-\gamma x_0^2)\} \) can always approach \( \phi_i + \pi_i \) when the state \( x_0 \) moves to zero, so that the instantaneous model may have some roots outside the unit circle to force the next state to increase. Thus, based on this definition of \( \gamma \) value, though it is not optimum, the EAR model is still assured to reveal the limit cycle behavior of the time series underlying.

Beneficial from this good idea, however, the best harvest is that the \( \gamma \) achieves to be quickly determined from original data set, accordingly, to real-time estimate the EAR model can be easily implemented because the estimate of other linear coefficients by the standard least squares (LS) is very quick for a personal computer.

4. Simulations

A nonlinear time series of limit cycle behavior containing 1000 data is artificially generated by the following EAR model to show the effectiveness of the proposal,

\[
x_t = (1.9281 + 0.1686e^{-0.2212x_{t-1}^2})x_{t-1} - (0.9380 + 0.1718e^{-0.2212x_{t-1}^2})x_{t-2} + \varepsilon_t
\]

where \( \{\varepsilon_t\} \) is the white noise with mean zero and variance 0.001. For this series, by the off-line estimating methods \(^{17}\), one can obtain very similar model as the original, while using the real-time approach proposed above (with \( \varepsilon = 0.0001 \)), the resulting model is as follows

\[
x_t = (1.9281 + 0.1686e^{-0.2212x_{t-1}^2})x_{t-1} - (0.9380 + 0.1718e^{-0.2212x_{t-1}^2})x_{t-2} + \hat{\varepsilon}_t
\]

where the variance of residual \( \{\hat{\varepsilon}_t\} \) is 0.0002. The estimated model looks like slightly changed in coefficients comparing to the original model, however, soon afterwards one can see that it still possesses the same characteristics of the original model. Encountered the check of the existence for a limit cycle, the original and estimated models are all satisfactory with conditions [a], [b] and [c], therefore limit cycle can be expected from the models. We also compare the limit cycle forms by plotting the two models' unforced response functions with two initial values arbitrarily selected as 1.0. From Fig. 1 it is obviously seen that the original and estimated models share the same form of limit cycle, therefore the two models are surely of the same dynamics though the difference in their appearance. Hence from this example, it is conclusive that the proposed method is suitable for the estimate of the EAR model.

![Fig. 1 The unforced responses of the original and estimated models for simulation data](image-url)
Table 1  The estimated model coefficients for "no chatter" series

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<td>-1.6471</td>
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<td>0.0989</td>
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<td>$\hat{\gamma}$</td>
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<td></td>
<td></td>
<td></td>
<td>2.8896</td>
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Table 2  The estimated model coefficients for "chatter" series

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<tr>
<td>$\hat{\gamma}$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>2.7620</td>
</tr>
</tbody>
</table>

(a) no chatter

(b) chatter

Fig. 2  Two original data sets recorded in a cutting experiment.

(a) no chatter

(b) chatter

Fig. 3  The unforced responses of two estimated models for cutting vibration data sets

5. Experiment Setup and data analysis

Some experiments were undertaken to investigate the effectiveness of chatter detecting in cutting process using the EAR model, where the vibration (acceleration) signal of the center in the thrust direction of the lathe is selected as the parameter to be monitored. Moreover, for the present study, some typical data sets measured at the different stages of chatter development in the cutting process are recorded, two of which are shown in Fig. 2. Each series has 512 samples, "no chatter" signal was recorded when cutting was in normal state, while "chatter" signal was recorded when chatter had been occurred. The cutting experiment was conducted with an AISI 1045 steel bar of 15.5 mm in diameter and 340 mm long. The experiment was started at a spindle speed of 900 rpm and a feed rate of 0.1 mm/rev. The depth of cut was kept constant at 1.5 mm throughout the cutting experiment.

To extract the feature contained in these two series of cutting vibration signal which had once been investigated by Shi and Aoyama, the EAR
model (1) of order \( p = 10 \) is used. The models of the two series are estimated by the above proposed real-time method (with \( \varepsilon = 0.0001 \)). The coefficients of two resulting models are shown in Tables 1 and 2, respectively. Then the check of existence for limit cycle is carried out. Expectedly, the model of “no chatter” series has not limit cycle but the model of “chatter” series has. Practically, as checking results, the model of “no chatter” series whose coefficients are shown in Table 1 does not satisfy the necessary condition [b] so that there is no limit cycle behavior expectable in this model; the model of “chatter” series whose coefficients are shown in Table 2 is satisfactory with all conditions [a], [b] and [c], therefore a limit cycle is existed in this model. Whether the models do have limit cycle or not is also checked by plotting the unforced response graphs of the two estimated models, as shown in Fig. 3, in which we see that the response function of the model of “no chatter” quickly converges to its stable singular point, but that of the model of “chatter” converges to a periodic oscillation. Hence the two EAR models really reveal the true properties of two series of cutting vibration signal.

Accordingly, on-line monitoring of machine tool chatter can be carried out with a step by step recording-modeling-limit cycle checking procedure, as proposed by Shi and Aoyama. Once the estimated model in the procedure is found having limit cycle, then chatter occurred can be warned to switch on some control devices in automatic manufacturing system. Note that the length of records used for modeling at each step should be determined before run, this study uses 512 data by balancing the trade-off of information contained and corresponding CPU time needed to calculation. Moreover, it has been found that the EAR model of order 10 is enough to fit the cutting vibration signal of 512 samples for limit cycle extraction.

6. Conclusion

As an effort to extract the characteristic feature of machine tool chatter not affected by the system working conditions and noise, the limit cycle behavior is proposed to be the index of chatter occurrence, and the EAR model is provided to detect the index. The present study also introduces a quickly estimating method of the EAR model for the objective of the implementation of the proposal in manufacturing system. The estimated EAR model is available for capturing the feature of time series underlying, which has been proved by the investigation to both simulation and practical cutting vibration signal. Note that, in the proposed procedure, calculating time is mainly consumed in the linear coefficients estimate, for which about \( 7(2p)^2 \times N \) arithmetic operations are needed if recursive least square is used, where \( N \) is the number of data (such as 512) and \( p \) is model order (such as 10). Therefore real-time implementation of the procedure can be guaranteed. Beneficial from this study, a reliable and lower cost on-line monitoring system of machine tool chatter is expected to achievement in the near future.

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References